Stagewise Implementations of Sequential Quadratic Programming for Model-Predictive Control

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Model Predictive Control (MPC) has become popular for online robot decision-making. It has shown compelling results with all kinds of robots ranging from industrial manipulators [1], quadrupeds [2]–[4] to humanoids [5], [6]. In robotics, Differential Dynamic Programming (DDP) [7] is a popular choice to solve OCPs because it exploits the problem's structure well. This advantage has led to a bustling algorithmic development over the past two decades [8]–[20]. In this work, we argue that this effervescence has hidden the fact that sparsity can be equally exploited by standard nonlinear optimization. Indeed, one might naively ask: why not use well-established optimization algorithms [21]? **Is there anything special in MPC that cannot be tackled by, for example, an efficient implementation of Sequential Quadratic Programming (SQP) [22]?** In this work, we show that special implementations of numerical methods developed by the optimization-based control community [23]–[26] are, in fact, sufficient to achieve state-of-the-art MPC on real robots.

Mayne first introduced DDP [7] as an efficient algorithm to solve nonlinear OCPs by iteratively applying a backward pass over the time horizon and a nonlinear forward rollout of the dynamics. This algorithm notably exhibits linear complexity in the time horizon and local quadratic convergence [27]. More recently, Todorov revived the interest in DDP by proposing the iterative Linear Quadratic Regulator (iLQR) [8], a variant discarding the second-order terms of the dynamics. It has since gained a lot of traction within the robotics community [3]–[5], [14], [18], and its similarity to Gauss-Newton optimization has been established [9], [28]. However, this approach faces two main limitations: 1) as a single shooting method, it requires a dynamically feasible initial guess, which makes the algorithm difficult to warm-start, an essential requirement to reduce computation times [12] and 2) enforcing equality and inequality constraints is not straightforward. The common practice is to enforce constraints softly using penalty terms in the cost function. But this approach is heuristic (i.e., it requires cost weight tuning) and tends to cause numerical issues [29].

Multiple shooting for optimal control, introduced in [30], addresses the first limitation: it accepts an infeasible initial guess. Several multiple shooting variants of DDP/iLQR were proposed in [12], [14] with significantly improved convergence abilities, which have enabled nonlinear MPC at high frequency on real robots [1], [3], [6], [14].

The second issue of enforcing constraints inside a DDP-like algorithm has been addressed in several works. [10] uses a DDP-based projected Newton method to bound control inputs. This approach has further been improved and deployed on a real quadruped robot in [17]. More recently, augmented Lagrangian methods have been used to enforce constraints in iLQR/DDP algorithms [11], [13], [16], [19]. However, their convergence behavior is less understood than DDP, whose seminal paper [7] was followed by sophisticated proofs [27]. To the best of our knowledge, it has not yet been shown that those recent DDP-based algorithms exhibit global convergence (i.e., convergence from any initial point to a stationary point) and quadratic local convergence.

In the face of these challenges, we propose to take a fresh look at the earlier literature. Indeed, Dunn et al. showed that Newton's method could also be implemented in a DDP-like fashion and equally benefit from linear complexity in the time horizon and quadratic convergence [31]. This finding indicates that optimal control does not fundamentally require new nonlinear optimization tools but only tailored implementations that exploit the time-induced sparsity structure. This naturally led to numerous extensions to the constrained case [22], [32]–[39]. This paradigm has since been applied to robotics: [26] recently proposed efficient software with an SQP implementation for OCP. [40] studied how to exploit the sparsity induced by time in IPOPT [41]. Unfortunately, this line of work has not benefited from as much experimental study as DDP-like algorithms. Recently, [42] showed impressive experimental results on quadrupeds using a tailored SQP implementation based on HPIPM [25], which lies in the continuity of previous works using [25], [26] on real hardware [43]–[45]. However, to the best of our knowledge, we are not aware of closed-loop constrained nonlinear MPC on torque-controlled robots.

We argue that there is **a gap between the optimization-based control and the robotics community**. On the one hand, an important part of the robotics community followed the successes of [46] and continued to propose DDP-like algorithms. On the other hand, the optimization-based control community followed the work of [22], [33] and proposed

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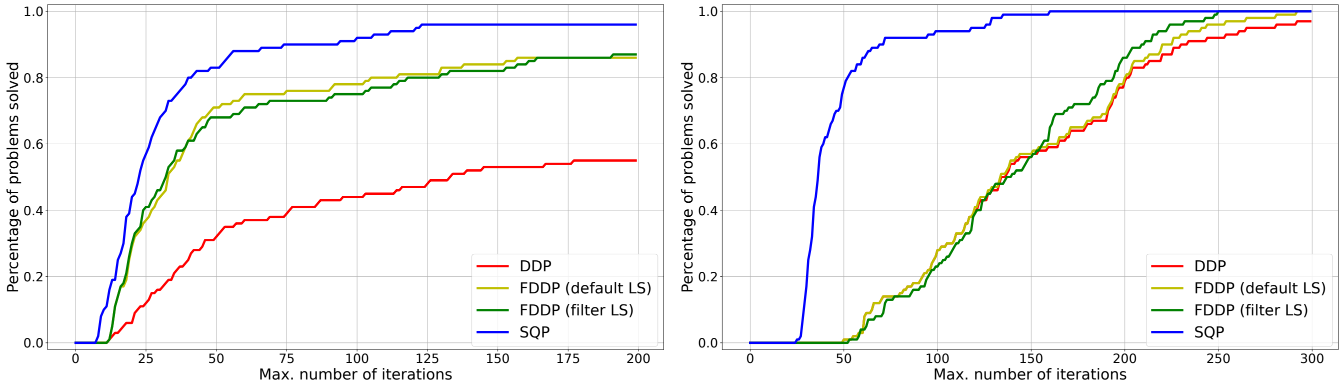

(a) Quadrotor pose task with randomized initial state.

(b) Humanoid taichi task with randomized end-effector goal.

Fig. 1: Percentage of problem solved as a function of the maximum number of iterations allowed on randomized unconstrained OCPs for various solvers: DDP, FDDP with default line-search, FDDP with filter line-search and our SQP. Our SQP implementation exhibits a faster and more robust convergence on difficult problems, such as the humanoid taichi task.

Fig. 2: Snapshots of a constrained end-effector tracking task with external disturbances.

efficient implementations of established optimization algorithms [24], [26], [40]. In this work, we aim to bridge this gap.

In this work, we follow the line of thought of the optimization-based control community in order to push the limits of closed-loop nonlinear MPC in robotics. First, we shed light on the direct connection between modern multiple-shooting DDP-like algorithms and textbook SQP algorithms. Second, we show through an experimental study that a standard stagewise SQP formulation is, in fact, superior to the state-of-the-art FDDP [14] (as illustrated in Figure 1). Third, we re-implement a QP solver tailored for optimal control by leveraging Riccati recursions in order to maintain linear complexity in the time horizon while also enforcing constraints [23], [47]. Using this custom QP implementation inside the SQP formulation, we can solve arbitrary nonlinear constrained OCPs efficiently while inheriting the well-known convergence properties of standard SQPs. Lastly, we demonstrate the ability of this SQP formulation to enforce arbitrary nonlinear constraints in MPC experiments on a torque-controlled manipulator (see Figure 2). To the best of our knowledge, this is the first demonstration of closed-loop nonlinear MPC with hard constraints on real hardware. The optimization software has been open-sourced in the `mim_solvers` library[1] to ensure reproducible experiments and enable easy use by the research community.

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