Designing Safe Quadrupedal Gaits

James Zhu, Selvin Garcia Gonzalez, Ardalan Tajbakhsh, and Aaron M. Johnson

I. Introduction

The robotics field has long had the goal of performing tasks in precarious work environments where human laborers are at risk of bodily harm. Recently, quadruped robots have begun to make this goal a reality, being deployed to serve areas such as construction sites [1], offshore electrical substations [2], and mine tunnels [3]. In these situations, robots may be required to navigate complex terrain such as steep slopes and narrow pathways. Not only do robots need to be able to plan feasible trajectories through these environments, but they must also do so safely in the presence of disturbances and errors. While many algorithms have been produced to improve robustness of legged locomotion [4–7], these methods still rely on non-trivial tuning of many parameters like contact sequence, contact timings, gait period, speed, etc.

There have been many investigations over several decades on how gait parameters such as step length, gait frequency, and duty factor affect the performance of locomotion [8–11]. One of the more well-known results that has been demonstrated on both horses [12] and quadrupedal robots [11,13] is that the walking gait is most efficient at low speeds while the trotting gait is advantageous at higher speeds. We find evidence that supports this idea (see Fig. 2a), but identify that duty factor may actually have a more direct effect on efficiency. We also extend the definition performance beyond just energy efficiency and examine the effect of speed and duty factor on stability.

Leveraging these insights, we aim to design quadrupedal gaits that are safer when navigating narrow pathways. Robots travelling through mine tunnels or crowded urban sidewalks may experience circumstances that constrain the stance width of their feet. Narrowing a legged robot's stance width shrinks the support polygon, which defines an area where the center of mass is easily stabilized. To counteract this deficiency, prior works have relied on inertial elements to stabilize the robot with the torso of the robot itself [14] or by augmenting the robot with a reaction wheel [15].

In this work, we explore two primary research questions:

- **RQ1: How do the gait parameters of duty factor and stance width relate to the stability performance of two common quadrupedal gaits: the walk and trot?**
- **RQ2: Given a stance width constraint, what combination of gait parameters enable safe traversal?**

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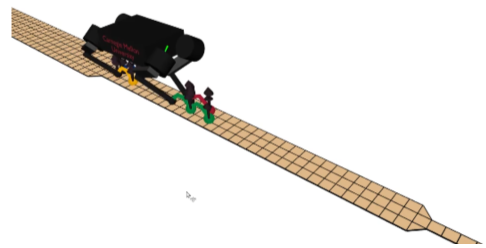


Fig. 1: Hierarchical locomotion planners like Quad-SDK [16] can generate plans to traverse perilous environments like narrow beams, but maintaining closed-loop stability in these scenarios remains an under-explored research question.

II. Hybrid Convergence Analysis

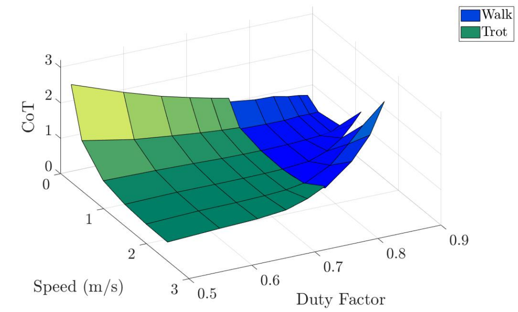
Legged robots are a type of hybrid system, where continuous domains (i.e. stance and flight phases) are linked by discontinuous hybrid events (i.e. foot touchdown and liftoff) [17]. Following our previous work in [4], the objective of this analysis is to track the evolution of errors over a hybrid trajectory and identify the worst-case direction of error expansion. This can be done by studying the linearized variational equations, which characterize the evolution of an error vector δx [18]. In continuous domains, the linearized variational equation over a discrete timestep is $\delta x_{i+1} \approx (A_\mathrm{I} - B_\mathrm{I} K_\mathrm{I})\delta x_i$ with A_I and B_I being the standard discretized dynamics derivatives and K_I linear feedback gains. Hybrid events have an analogous operator called the saltation matrix $\Xi_{(\mathrm{I},\mathrm{J})}$ which maps error from before a hybrid event occurring at $\delta x(t^-)$ to after $\delta x(t^+)$, such that $\delta x(t^+) \approx \Xi_{(\mathrm{I},\mathrm{J})}\delta x(t^-)$ [19].

Consider a trajectory beginning at state $x_0 = x(t_0)$ at time t_0 executed until time t_f at final state x_f. Define the fundamental solution matrix, Φ, which maps an initial error vector to its evolution at time t_f such that $\delta x_f \approx \Phi \delta x_0$ [20]. Φ can be computed by composing the linearized variational equations in each continuous domain ($\tilde{A} := A - BK$) and the saltation matrices (Ξ) at each hybrid event [21], such that for a hybrid trajectory with N domains, the fundamental solution matrix is $\Phi = \tilde{A}_N \Xi_{(N-1,N)} \dots \Xi_{(2,3)} \tilde{A}_2 \Xi_{(1,2)} \tilde{A}_1$ [4].

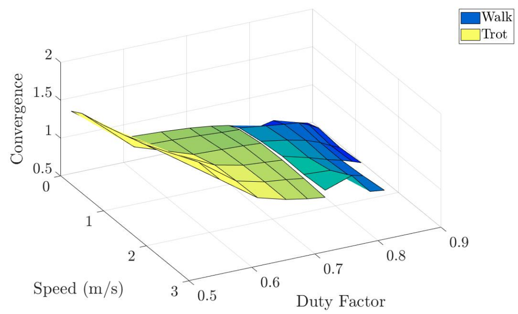
Let χ be the convergence measure [4], which is the largest singular value of Φ. Smaller values of χ correspond to an improvement in worst-case error expansion, with $\chi < 1$ indicating all errors will shrink over the trajectory.

III. Methods

Based on our research questions, we establish the following hypotheses:

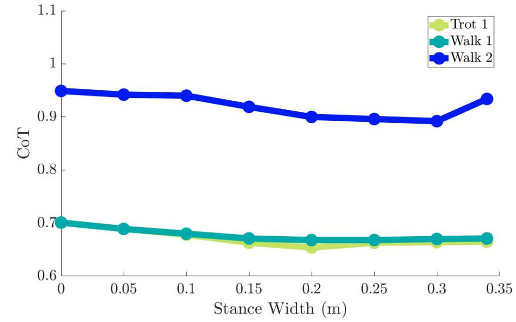


(a)

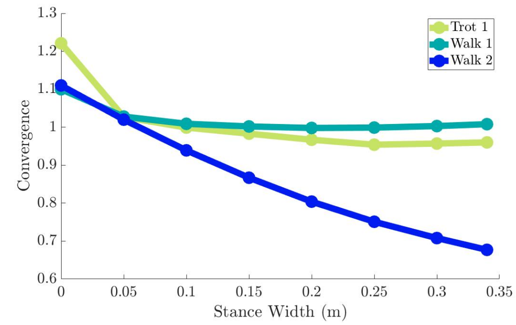


(b)

Fig. 2: The walking and trotting gaits transition quite smoothly between their natural duty factor regimes. The combination of duty factor and speed is a strong predictor of a gait's energy efficiency, while convergence is primarily correlated with duty factor. Note for both CoT and Convergence metrics, a lower value indicates better performance.



(a)



(b)

Fig. 3: As stance width narrows, convergence generally worsens, while efficiency does not differ significantly. The performance of walking and trotting gaits with equivalent speed and duty factor remain similar. There is also indication of a trade-off between sacrificing efficiency to improve convergence, at least up until a stance width of about 0.05.

- *Hypothesis 1:* Increasing the duty factor of a gait will improve the convergence
- *Hypothesis 2:* Narrowing stance width will worsen convergence
- *Hypothesis 3:* Overall, the walking gait will tend to be more convergent than the trotting gait

To evaluate these hypotheses, we wrote a custom direct collocation trajectory optimization method using Pinocchio [22] to compute the continuous and hybrid dynamics and the corresponding dynamics derivatives. We chose a direct collocation method because it allows us to explicitly define the contact timings and therefore the duty factor of each trajectory. The cost function consists of standard quadratic costs on input torques and some shaping costs on the body and leg trajectories and a periodicity constraint is imposed. For every trajectory, an LQR controller is generated with constant weights on state and input. The model used was the Ghost Spirit 40 quadruped.

We performed two experiments. First, we generated walking and trotting gaits at 6 speeds $(0.25, 0.5, 1, 1.5, 2, 2.5)$ m/s and 6 duty factors $(0.75, 0.792, 0.821, 0.844, 0.861, 0.875$ for the walk and $0.5, 0.583, 0.643, 0.688, 0.722, 0.75$ for the trot). For each trajectory, we computed the energy efficiency (positive work plus Ohmic motor losses over distance) and convergence measure and plotted the results in Fig. 2.

In the second experiment, we selected 3 gaits that exhibited good efficiency and convergence properties: 1) the 1.5 m/s, 0.75 duty factor trot (Trot 1); 2) the 1.5 m/s, 0.75 duty factor walk (Walk 1); and 3) the 1 m/s, 0.875 duty factor walk (Walk 2). We then generated trajectories with each of these speed/duty factor combinations while constraining the gaits to narrowing stance widths of $(0.34, 0.3, 0.25, 0.2, 0.15, 0.1, 0.05, 0)$ m.

IV. Results

The results of Experiment 1 are shown in Fig. 2, where there is a nearly smooth transition between the two gaits at 0.75 duty factor. We can see that while the walk is generally more efficient at lower speeds and the trot at higher speeds, duty factor is a much more direct predictor of efficiency, with lower duty factors being more efficient at higher speeds and vice versa. Fig. 2b shows that speed does not have a strong relationship to convergence and that increasing duty factor overall improves convergence. Experiment 2 found that the efficiency and convergence of the gaits with equivalent speeds and duty factors maintain their similarity as stance width narrows. Fig. 3 shows that we can trade-off efficiency and convergence for narrow paths, up until stance width of about 0.05 m. These results indicate support for Hypotheses 1 and 2, while providing evidence against Hypothesis 3.

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